KOEFFITSIYENTLAR NAZARIYASI

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Annotatsiya: *Trigonometrik ketma-ketlik yordamida trigonometriya koeffitsiyentini aniqlash, aylana yoyining uzunligi yordamida yoy koeffitsiyentini aniqlash va uning trigonometriya koeffitsiyentiga bog'lanishini o'rganish.*

Trigonometrik ketma-ketlik yordamida trigonometriya koeffitsiyentini aniqlash. Dastlab ikkita mustaqil A va B trigonometrik ketma-ketlik tasvirlangan to'plamlarni olamiz:

$$A = \{\sin x, \sin 2x, \sin 3x, \dots, \sin((n-1)x), \sin(nx)\}$$
$$B = \{\cos x, \cos 2x, \cos 3x, \dots, \cos((n-1)x), \cos(nx)\}$$

Shartli ravishda A to'plam elementlari yig'indisini X, B to'plam elementlari yig'indisini Y sifatida belgilaymiz:

$$X = \sum_{i=1}^{n} \sin(ix)$$
$$Y = \sum_{i=1}^{n} \cos(ix)$$

X va Y ning mos qonuniyatlarini topish jarayonini parallel olib borish uchun esa ifodalarni sistemada birlashtiramiz:

$$\begin{cases} X = \sin x + \sin 2x + \sin 3x + \dots + \sin((n-1)x) + \sin(nx) \\ Y = \cos x + \cos 2x + \cos 3x + \dots + \cos((n-1)x) + \cos(nx) \end{cases}$$

O'z navbatida, quyidagi qonuniyatlardan foydalanib, $\sin 2x$ (yoki $\cos 2x$) dan $\sin(nx)$ (yoki $\cos(nx)$) gacha topib olamiz:

$$\sin(nx) = \sin((n-1)x)\cos x + \sin x\cos((n-1)x)$$
$$\cos(nx) = \cos((n-1)x)\cos x - \sin x\sin((n-1)x)$$

Topib olingan ifodalarni sistemaga qo'yib, soddalashtiramiz va natijada quyidagi sistemaga ega bo'lamiz:

$$\begin{cases} X = \cos x(X - \sin(nx)) + \sin x(Y + 1 - \cos(nx)) \\ Y = \cos x(Y + 1 - \cos(nx)) - \sin x(X - \sin(nx)) \end{cases}$$

Qavslarni ochib chiqamiz va X (yoki Y) ning Y (yoki X) ga quyidagicha bog'langani ma'lum bo'ladi:

$$\begin{cases} X = \dfrac{Y\sin x + \sin x - \sin x\cos(nx) - \sin(nx)\cos x}{1 - \cos x} \\ Y = \dfrac{\cos x - \cos x\cos(nx) - X\sin x + \sin x\sin(nx)}{1 - \cos x} \end{cases}$$

Nihoyat o'rniga qo'yish yo'li bilan quyidagi X (yoki Y) ning Y (yoki X) ga bog'lanmagan, ya'ni erkli qonuniyatlarini aniqlaymiz:

$$\begin{cases} X = \frac{1}{2}\left(\frac{\sin x\left(1-\cos(nx)\right)}{1-\cos x} + \sin(nx)\right) \\ Y = \frac{1}{2}\left(\frac{\sin x \sin(nx)}{1-\cos x} - 1 + \cos(nx)\right) \end{cases}$$

E'tibor bersangiz, har ikkala qonuniyat uchun umumiy bo'lgan bir ifoda mavjud. Aynan ushbu ifodani T_x ko'rinishida belgilab, trigonometriya koeffitsiyenti deb atashni taklif qilaman.

Trigonometriya koeffitsiyenti, o'z navbatida, quyidagidir:

$$T_x = \frac{\sin x}{1-\cos x} = \frac{1+\cos x}{\sin x}$$

Demak, A va B to'plam elementlari uchun quyidagi qonuniyatlar o'rinlidir:

$$\sum_{i=1}^{n} \sin(ix) = \frac{\sin(nx)}{2}\left(\frac{T_x}{T_{nx}} + 1\right)$$

$$\sum_{i=1}^{n} \cos(ix) = \frac{\sin(nx)}{2}\left(T_x - \frac{1}{T_{nx}}\right)$$

Bu yerda, trigonometriya koeffitsiyenti indeksidagi x trigonometriya koeffitsiyentining tartibini bildiradi va u radianda o'lchanadi. Misol uchun, $90°$ li trigonometriya koeffitsiyenti 90-tartibli emas, balki $\frac{\pi}{2}$-tartibli trigonometriya koeffitsiyenti hisoblanadi.

Trigometriya koeffitsiyenti uchun quyidagi tengliklar o'rinlidir:

$$T_{x+y} = \frac{T_x T_y - 1}{T_x + T_y}$$

$$T_{x-y} = \frac{T_x T_y + 1}{T_y - T_x}$$

$$T_x = \frac{T_{x+y}T_{x-y} + 1 + \sqrt{\left({T_{x+y}}^2 + 1\right)\left({T_{x-y}}^2 + 1\right)}}{\left(T_{x-y} - T_{x+y}\right)}$$

$$T_y = \frac{T_{x+y}T_{x-y} - 1 + \sqrt{\left({T_{x+y}}^2 + 1\right)\left({T_{x-y}}^2 + 1\right)}}{T_{x+y} + T_{x-y}}$$

Aylana yoyining uzunligi yordamida yoy koeffitsiyentini aniqlash. Aylana shaklida, tugun ko'rinishidagi berk, cho'zilmas ipni tasavvur qilamiz va uni to'g'ri burchakli uchburchak ko'rinishiga keltiramiz. Bu holatda ipning uzunligi o'zgarmasligi hisobidan to'g'ri burchakli uchburchakning perimetri aylananing perimetriga tengligi hosil bo'ladi:

$$a + b + c = 2\pi r$$

Dastlab katetlardan birini radiusga karrali ifodaga tenglaymiz:

$$a = M_x r$$

Bu yerda, M_x – aylana koeffitsiyenti.

Yuqoridagi ifodalarni Pifagor teoremasi bilan sistemada birlashtiramiz:

$$\begin{cases} a + b + c = 2\pi r \\ a^2 + b^2 = c^2 \end{cases}$$

$$\begin{cases} c + b = r(2\pi - M_x) \\ c^2 - b^2 = {M_x}^2 r^2 \end{cases}$$

$$\begin{cases} c + b = r(2\pi - M_x) \\ c - b = \dfrac{{M_x}^2 r}{2\pi - M_x} \end{cases}$$

Yuqoridagi tenglikdan b va c ni topib olamiz:

$$b = \frac{2\pi r(\pi - M_x)}{2\pi - M_x}$$

$$c = \frac{r\left({M_x}^2 - 2\pi M_x + 2\pi^2\right)}{2\pi - M_x}$$

To'g'ri burchakli uchburchak qonuniyatlariga muvofiq, quyidagi ifodalarni hosil qilamiz:

$$\sin x = \frac{a}{c} = \frac{M_x(2\pi - M_x)}{{M_x}^2 - 2\pi M_x + 2\pi^2}$$

$$\cos x = \frac{b}{c} = \frac{2\pi(\pi - M_x)}{{M_x}^2 - 2\pi M_x + 2\pi^2}$$

Yoy koeffitsiyenti uchun quyidagi tengliklar o'rinlidir:

$$M_{x+y} = \frac{2\pi\left(\pi M_x + \pi M_y - M_x M_y\right)}{2\pi^2 - M_x M_y}$$

$$M_{x-y} = \frac{2\pi^2\left(M_x - M_y\right)}{M_x M_y - 2\pi M_y + 2\pi^2}$$

$$M_x = \frac{\pi\left(2\pi^2 - 2\pi M_{x-y} + M_{x+y} M_{x-y} - M\right)}{2\pi^2 - \pi M_{x+y} - \pi M_{x-y} + M_{x+y} M_{x-y}}$$

$$M_y = \frac{2\pi^2 - M_{x+y} M_{x-y} - M}{2\pi - M_{x+y} - M_{x-y}}$$

Bu yerda, M quyidagiga teng:

$$M = \sqrt{\left({M_{x+y}}^2 - 2\pi M_{x+y} + 2\pi^2\right)\left({M_{x-y}}^2 - 2\pi M_{x-y} + 2\pi^2\right)}$$

Yoy koeffitsiyentining trigonometriya koeffitsiyentiga bog'lanishini o'rganish. Dastlab trigonometriya koeffitsiyentidagi $\sin x$ va $\cos x$ ning o'rniga ularning yoy koeffitsiyenti yordamida ifodalangan formulasini qo'yib olamiz:

$$T_x = \frac{1 + \cos x}{\sin x} = \frac{{M_x}^2 - 4\pi M_x + 4\pi^2}{2\pi M_x - {M_x}^2}$$

Endi bu ifodani ko'paytuvchilarga ajratib qisqartirib olamiz:

$$T_x = \frac{(2\pi - M_x)^2}{M_x(2\pi - M_x)} = \frac{2\pi - M_x}{M_x}$$

Xuddi shu ifodadan yoy koeffitsiyentini topib olamiz:

$$M_x = \frac{2\pi}{1 + T_x}$$